

Name: _____

Student ID: _____

Instructions:

1. This exam has 5 pages. Please make sure you have all pages.
2. The point value of each problem occurs to the left of the problem.
3. **You must show correct work to receive credit.** Correct answers with inconsistent work or with no justification will not be given credit.
4. Only non-graphing and non-programmable calculators are allowed.
5. **Turn off and put away all cell phones.**

Page	Points	Points Possible
2		14
3		14
4		14
5		8
Total		50

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1. Let $A = \begin{bmatrix} 4 & 2 & 3 & 1 \\ 1 & 0 & -3 & -1 \\ 6 & 4 & 6 & 2 \\ 8 & 4 & 3 & 1 \end{bmatrix}$.

(a) (6 pts) Find the reduced row echelon form of A .

(b) (4 pts) Solve the system $A\mathbf{x} = \mathbf{0}$.

(c) (4 pts) If \mathbf{y} is a solution of $A\mathbf{x} = \mathbf{b}$, find another solution of $A\mathbf{x} = \mathbf{b}$.

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2. (4 pts) If A and B are 2×2 matrices with $\det(A) = 3$ and $\det(B) = 2$, find $\det(2A^2(3B)^{-1}A^t)$.

3. (10 pts) Find all values of λ for which the system $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ \lambda & 2 & 0 & 1 \\ \lambda^2 & 1 & 0 & \lambda \end{array} \right]$ is consistent.

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4. (6 pts) Write the polynomial $p(x) = x^3 - 2x^2 + 4x - 7$ as a linear combination of

$$p_1(x) = x^2 - 2x + 2, \quad p_2(x) = x^3 + x^2 - 1, \quad p_3(x) = x^2 - x + 2.$$

5. (8 pts) Let $\mathbf{v}_1, \dots, \mathbf{v}_k$ be linearly independent vectors in \mathbb{R}^n , and suppose that A is an invertible $n \times n$ matrix. Define vectors $\mathbf{w}_i = A\mathbf{v}_i$, for $i = 1, \dots, k$. Show that the vectors $\mathbf{w}_1, \dots, \mathbf{w}_k$ are linearly independent.

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6. (8 pts) Let A be a 5×5 matrix and let B be the matrix obtained from A by multiplying row 1 by 3. Prove that $\det(B) = 3 \det(A)$.